Integration Techniques Calculus of a Single Variable X. Du

- **u-substitution** (change of variables): Sometimes, if $\int f(x)dx$ is not easily differentiable at it involves a composite function, you may find a function $u(x)$ with a suitable $u'(x)$ to substitute in for *x* such that $\int f(x)dx = \int f(u)\frac{dx}{du}du$. Think of this as doing the chain rule backwards (i.e. $\int f'(g(x))g'(x)dx = f(g(x)) + C$). However, change of variables can also be used in settings which do not involve a composite function, in which you will replace all forms of *x* with *u*.
- Rewrite the integrand using logarithmic or exponential properties.
  - Logs: expand the log and evaluate each term. Base change to *e* if necessary.
  - Exponents: Rewrite in terms of base *e*. Ex: $a^x = e^{x\ln a}$
- Rewrite the integrand using trigonometric identities and definitions.
  - Pythagorean identities and double angle identities come in handy very well.
- Rewrite hyperbolic functions using exponents.
- **Integration by Parts**: $\int udv = uv - \int vdu$
  - Derived from product rule: $(uv)' = udv + vdu$
  - Order of sequence for determining *u*: LIATE (logs, inv trig, algebraic, trig, exp)
  - Sometimes, you may have to do integration by parts multiple times. Use table!
  - If you get a loop, move it to the other side and multiply by the reciprocal.
- **Trigonometric Substitution**
  - You cannot use an easier method.
  - Substitute a trigonometric function in for *x* and its derivative for *dx*.
  - If the integrand involves $u^2 - (x-a)^2$, use $x = a + u\sin\theta$ as your substitution.
  - If the integrand involves $u^2 + (x-a)^2$, use $x = a + u\tan\theta$ as your substitution.
  - If the integrand involves $(x-a)^2 - u^2$, use $x = a + u\sec\theta$ as your substitution.
  - Use if you have one over a quadratic that cannot be factored - complete the square and use trig sub.
- **Partial Fractions**
  - Rewrite fraction as a sum of individual partial fractions.
  - Use systems of equations to solve for unknowns.
  - If one factor has a multiplicity greater than one, you must list a series of powers up to that power.
  - If one factor is a quadratic, you must put a linear factor in the numerator.
- **Improper Integrals**
  - Either at least one bound of the function is infinite or at one of the bounds, the antiderivative is infinite.
  - Use a limit to evaluate.
- **Make the integrand complex** (sounds counterintuitive, but it works!)
  - Let $f(x)$ be a complex function.
  - $\int \mathrm{Re}[f(x)]dx = \mathrm{Re}\left[\int f(x)dx\right]$ $\int \mathrm{Im}[f(x)]dx = \mathrm{Im}\left[\int f(x)dx\right]$